ECE 343: Signals and Systems

Homework 6: Fourier Series

For all of these problems, don't worry about reducing the answer down completely. Things like $\cos(\pi k)$ don't need to be reduced to $(-1)^k$.

1. Periodicity

   (a) Let $x(t) = 2\cos^2(2t) + \sin(\frac{t}{5})$. Find the period of x and the Fourier series coefficients $X[k]$.

   (b) Repeat part (a) with $y(t) = j\cos(\sqrt{2}t) + \sin(2t)$.

2. Half wave symmetry

   (a) Suppose that a T_0-periodic function $x(t)$ satisfies the equation $x(t - \frac{T_0}{2}) = -x(t)$. This means that for the second half of the period, x looks the same as the first half, only negated. Show that

   $$X[k] = \begin{cases} \frac{2}{T_0}\int_0^{T_0/2} x(t)e^{-2\pi jkt/T_0}dt & k \text{ odd} \\ 0 & k \text{ even} \end{cases}$$

   (b) Find the Fourier series coefficients of the functions plotted in Figure P6.1-5 (p. 671) of the text.

3. Find the Fourier series coefficients of a square wave of duty cycle α. In other words, find $X[k]$ for

   $$x(t) = \begin{cases} 1 & 0 < t < \frac{\alpha}{T_0} \\ 0 & \frac{\alpha}{T_0} < t < T_0 \end{cases}, \quad 0 < \alpha < 1$$

   $$x(t - T_0) = x(t)$$

4. Imagine that we have an ideal low pass filter that passes frequencies less than 10 Hz unchanged, but will not allow any frequency above 10 Hz through. This filter is an LTI system.

   (a) What is the impulse response $h(t)$ and the transfer function $H(f) = \mathcal{F}\{h(t)\}$?

   (b) Suppose we input a 50% duty cycle, 3 Hz square wave into this filter. Find *and plot* the output $y(t)$.

   $$x(t) = \begin{cases} 1 & |t| < \frac{1}{12} \\ 0 & -\frac{1}{6} < t < -\frac{1}{12} \\ 0 & \frac{1}{12} < t < \frac{1}{6} \end{cases}$$

   $$x(t + \frac{1}{3}) = x(t)$$